

# Correcting a proof in the InfoGAN paper\*

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May 2018

The InfoGAN paper<sup>1</sup> has the following lemma:

**Lemma 5.1.** For random variables  $X, Y$  and function  $f(x, y)$  under suitable regularity conditions:  $\mathbb{E}_{x \sim X, y \sim Y|x}[f(x, y)] = \mathbb{E}_{x \sim X, y \sim Y|x, x' \sim X|y}[f(x', y)]$ .

The statement is correct, but the proof in the paper is confused – here’s a step where  $x$  mysteriously becomes  $x'$ :

$$\begin{aligned} & \int_x \int_y P(x, y) f(x, y) \int_{x'} P(x'|y) dx' dy dx \\ &= \int_x P(x) \int_y P(y|x) \int_{x'} P(x'|y) f(x', y) dx' dy dx \end{aligned}$$

After consulting with others, we weren’t able to understand that step of the proof. Instead, Nic Ford found this alternative proof. Hopefully this could help others reading the paper.

*Proof.*

$$\begin{aligned} \mathbb{E}_{x \sim X, y \sim Y|x}[f(x, y)] &= && \text{make expectations explicit...} \\ \mathbb{E}_{x \sim P(X)} [\mathbb{E}_{y \sim P(Y|X=x)}[f(x, y)]] &= && \text{by definition of } P(Y|X=x)\dots \\ \mathbb{E}_{x, y \sim P(X, Y)}[f(x, y)] &= && \text{by definition of } P(X|Y=y)\dots \dots \\ \mathbb{E}_{y \sim P(Y)} [\mathbb{E}_{x \sim P(X|Y=y)}[f(x, y)]] &= && \text{rename } x \text{ to } x'\dots \\ \mathbb{E}_{y \sim P(Y)} [\mathbb{E}_{x' \sim P(X|Y=y)}[f(x', y)]] &= && \text{by the law of total expectation...} \\ \mathbb{E}_{x \sim P(X)} [\mathbb{E}_{y \sim P(Y|X=x)} [\mathbb{E}_{x' \sim P(X|Y=y)}[f(x', y)]]] &= && \text{make expectations implicit...} \\ \mathbb{E}_{x \sim X, y \sim Y|x, x' \sim X|y}[f(x', y)] & & & \end{aligned}$$

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\*Read and comment on the latest version of this note at <http://aoliver.org/correct-proof-of-infogan-lemma>

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<sup>1</sup><https://arxiv.org/pdf/1606.03657.pdf>